Status of Cosmological Parameters: $\Omega_0 \approx 0.3$ vs. $\Omega = 1$ [*]

Joel R. Primack

Santa Cruz Institute for Particle Physics,
University of California, Santa Cruz, CA 95064

Abstract

The cosmological parameters that I will discuss are the traditional ones: the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}, the age of the universe t_0, the average density $\Omega_0 \equiv \bar{\rho}/\rho_c$ in units of critical density ρ_c, and the cosmological constant Λ (or $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$). To focus the discussion, I will concentrate on the issue of the value of Ω_0 in currently popular cosmological models in which most of the dark matter is cold, especially Cold + Hot Dark Matter (CHDM), and flat ($\Omega_0 + \Omega_\Lambda = 1$) low-$\Omega$ CDM with a Cosmological Constant (ΛCDM). The evidence would favor a small $\Omega_0 \approx 0.3$ if (1) the Hubble parameter actually has the high value $h \approx 0.75$ favored by many observers, and the age of the universe $t_0 \geq 13$ Gy; or (2) the baryonic/total mass ratio in clusters of galaxies is actually $\sim 15\%$, about 3 times larger than expected for standard Big Bang Nucleosynthesis in an $\Omega = 1$ universe, and standard BBN is actually right in predicting that the density of ordinary matter is $\Omega_b \approx 0.0125h^{-2}$, based mainly on ^{4}He and 7Li data, despite the recent measurement by Tytler of $D/H = 2.4 \times 10^{-5}$ in two high-redshift Lyman limit systems, implying $\Omega_b \approx 0.024h^{-2}$. The evidence would favor $\Omega = 1$ if (1) the POTENT analysis of galaxy peculiar velocity data is right, in particular regarding outflows from voids or the inability to obtain the present-epoch non-Gaussian density distribution from Gaussian initial fluctuations in a low-Ω universe; or (2) the preliminary report from LSND indicating a neutrino mass $\gtrsim 2.4$ eV is right, since that would be too much hot dark matter to allow significant structure formation in a low-Ω_0 ΛCDM model. Statistics on gravitational lensing of quasars provide a strong upper limit on Λ. It also appears to be possible to measure the deceleration parameter $q_0 = \Omega_0/2 - \Omega_\Lambda$ on very large scales using high-redshift Type Ia supernovae; the preliminary results suggest that $\Omega_0 \sim 1$ and Ω_Λ is small. The era of structure formation is another important discriminant between these alternatives, low Ω_0 favoring earlier structure formation, and $\Omega = 1$ favoring later formation with many clusters and larger-scale structures still forming today. Reliable data on all of these issues is becoming available so rapidly today that there is reason to hope that a clear decision between these alternatives will be possible within the next few years.

1

1 Introduction

As I write this in spring 1996 [1], there is still concern about a crisis in cosmology. The first article [2] using HST observations of Cepheid variable stars to determine a distance to a relatively distant galaxy, the beautiful face-on spiral M100, was published about a year ago. The distance obtained was 17.1 ± 1.8 Mpc. With the additional assumptions that M100 lies in the core of the Virgo cluster and that the recession velocity of Virgo corrected for infall is about 1400 km s^{-1}, the value obtained for the Hubble parameter is at the high end of recent estimates: $H_0 = 80 \pm 17 \,\mathrm{km\,s^{-1}\,Mpc}$. Using $h = 0.8$ gives, for $\Omega = 1$ and a vanishing cosmological constant $\Lambda = 0$, a very short age for the universe $t_0 = 8.15$ Gyr, almost certainly younger than the ages of Milky Way globular clusters and even some nearby white dwarfs. Even with $\Omega_0 = 0.3$, about as low as permitted by observations, and with $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 0.7$, perhaps even higher than current data allow, $t_0 = 11.8$ Gyr for $h = 0.8$, which is also uncomfortably short. Is this a crisis? Does it undermine the strong evidence for the standard Big Bang? I don't think so. Given the considerable uncertainties reflected in the large quoted error on H_0, I think even $\Omega = 1$ models are not excluded. But this Cepheid measurement of the distance to M100 bodes well for the success of the HST Key Project on the Extragalactic Distance Scale, which seeks to measure H_0 to 10% within a few years. There has also been recent progress in using Type Ia supernovae as distance indicators, for measuring both H_0 and the deceleration parameter $q_0 = \Omega_0/2 - \Omega_\Lambda$. The expectation that accurate measurements of the key cosmological parameters will soon be available is great news for theorists trying to construct a fundamental theory of cosmology, and helps motivate the present summary.

In addition to the Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}, I will discuss the age of the universe t_0, the average density Ω_0, and the cosmological constant Λ. But there are several additional cosmological parameters whose values are critical for modern theories: the densities of ordinary matter Ω_b, cold dark matter Ω_c, and hot dark matter Ω_ν, and, for primordial fluctuation spectra $P(k) = Ak^{n_p}$, the index n_p and the amplitude A, or equivalently (for a given model) the bias parameter $b \equiv 1/\sigma_8$, where $\sigma_8 \equiv (\delta M/M)_{rms}$ on a scale of $8\,h^{-1}$ Mpc. A full treatment of these parameters would take a much longer article than this one, so to focus the discussion I will concentrate on the issue of the value of the density Ω_0 in currently popular cosmological models in which most of the dark matter is cold. Although much of the following discussion will be quite general, it will be helpful to focus on two specific cosmological models which are perhaps the most popular today of the potentially realistic models: low-Ω Cold Dark Matter with a Cosmological Constant (ΛCDM, discussed as an alternative to $\Omega = 1$ CDM since the beginning of CDM [3,4], and worked out in detail in [5]), and $\Omega = 1$ Cold + Hot Dark Matter (CHDM, proposed in 1984 [6], and first worked out in detail in 1992-3 [7,8]). I will begin by summarizing the rationale for these models.

2   Models with Mostly Cold Dark Matter

Let me begin here by recalling the definitions of "hot" and "cold" dark matter. These terms describe the astrophysically relevant aspects of candidate dark matter particles. The fact that the observational lower bound on Ω_0 — namely $0.3 \lesssim \Omega_0$ — exceeds the most conservative upper limit on baryonic mass $\Omega_b \lesssim 0.03h^{-2}$ from Big Bang Nucleosynthesis [9] is the main evidence that there must be such nonbaryonic dark matter particles.

About a year after the big bang, the horizon surrounding any point encompassed a mass of about $10^{12}M_\odot$, the mass now in the dark matter halo of a large galaxy like the Milky Way. The temperature then was about a kilovolt. We define *cold* dark matter as particles that were moving sluggishly, and *hot* dark matter as particles that were still relativistic, at that time. As Kim Griest and Antonio Masiero discussed in their Varenna lectures, the lightest superpartner particle (LSP neutralino) and the axion remain the best motivated cold dark matter candidates, although of course many other possibilities have been suggested.

The three known neutrino species ν_e, ν_μ, and ν_τ are the standard hot dark matter candidates. Their contribution to the cosmological density today is

$$\Omega_\nu = \frac{\sum_i m(\nu_i)}{94h^2 \, \text{eV}}$$

Since $\Omega_\nu < \Omega_0 \lesssim 2$, each neutrino's mass must be much less than a keV, so they were certainly moving at relativistic speeds a year after the big bang. Any of these neutrinos that has a cosmologically significant mass ($\gtrsim 1$ eV) is therefore a hot dark matter particle.

If a horizon-sized region has slightly higher than average density at this time, cold dark matter — moving sluggishly — will preserve such a fluctuation. But light neutrinos — moving at nearly the speed of light — will damp such fluctuations by "free streaming." For example, two years after the big bang, the extra neutrinos will have spread out over the now-larger horizon. The smallest fluctuations that will not suffer this fate are those that come into the horizon when the neutrinos become nonrelativistic, i.e. when the temperature drops below the neutrino mass. In a universe in which most of the dark matter is hot, primordial fluctuations will damp on all scales up to superclusters (with mass $\sim 10^{16}M_\odot$), leading to a sequence of cosmogony (cosmological structure formation) in which galaxies form only after superclusters. But this is contrary to observations, which show galaxies to be old but superclusters still forming. Indeed, with fluctuations on large scales consistent with the COBE upper limit, standard HDM models (i.e. with the dark matter being mostly neutrinos, and a Zel'dovich spectrum of Gaussian adiabatic fluctuations) cannot form a significant number of galaxies by the present. Thus most current comparisons of cosmological models with observations have focused on models in which most of the dark matter is cold.

The standard CDM model [3] assumed a Zel'dovich (i.e. $n_p = 1$) spectrum of primordial Gaussian adiabatic fluctuations with $\Omega = 1$. It had the great virtues of simplicity and

predictive power, since it had only one free parameter, the amplitude or bias b. Moreover, for a while it even looked like it agreed with all available data, with $b \approx 2.5$. One early warning that all was not well for CDM was the cosmic background dipole anisotropy, indicating a large velocity of the local group with respect to the cosmic background radiation rest frame, about 600 km s^{-1}. I confess that I and many other theorists did not immediately appreciate its possibly devastating impact. However, as evidence began to accumulate, starting in 1986, that there were large-scale flows of galaxies with such velocities [10], it became clear that standard CDM could fit these large-scale galaxy peculiar velocities (i.e. motions in addition to the general Hubble expansion) only for $b \approx 1$. Standard CDM had various problems for any value of b; for example, the CDM matter correlation function, and hence also the galaxy and cluster correlations, are negative on scales larger than about $30\,h^{-1}\,$Mpc, while observations on these large scales show that the cluster correlations are at least $\sim 3\sigma$ positive [11]. A low value of the bias parameter subsequently also turned out to be required by the COBE DMR data, which was first announced in April 1992. But for such a small $b \lesssim 1$, CDM produces far too many clusters and predicts small-scale galaxy velocities that are much too large [12]. Thus standard CDM does not look like a very good match to the now-abundant observational data. But it did not miss by much: if the bias parameter b is adjusted to fit the COBE data, the fluctuation amplitude is too large on small scales by perhaps a factor of $\sim 2 - 3$.

In the wake of the discovery of the existence of large-scale galaxy peculiar velocities, I suggested that Jon Holtzman (then a UCSC graduate student whose planned Ph.D. research based on HST observations had been indefinitely postponed by the Challenger explosion) improve the program that George Blumenthal and I had written to do linear CDM calculations, and use it to investigate a variety of models in which the dark matter was mostly cold. He ultimately worked out a total of 94 such models, about half of them including some hot dark matter, and (since this was the largest such suite of interesting models all worked out the same way) his thesis [13] provided the basis for the COBE-DMR interpretation paper [14]. Meanwhile, in a follow-up paper [15], we showed that of all these CDM-like models the ones that best fit the available data — especially the cluster correlations — were $\Omega = 1$ Cold + Hot Dark Matter (CHDM), and low-Ω Cold Dark Matter with a Cosmological Constant (ΛCDM). Since both of these models turned out to fit all available data rather well when their fluctuation amplitudes were normalized to COBE observations, they remain perhaps the most popular models for galaxy formation and large scale structure. Moreover, since CHDM works best for $h \approx 0.5$ while ΛCDM works best for higher h, they will serve nicely for this review as representatives of these two opposing alternatives.

3   Age of the Universe t_0

The strongest lower limits for t_0 come from studies of the stellar populations of globular clusters (GCs). Standard estimates of the ages of the oldest GCs are 14-18 Gyr [16], and

a conservative lower limit on the age of GCs is 13 ± 2 Gyr [17], which is then a lower limit on t_0. The main uncertainty in the GC age estimates comes from the uncertain distance to the GCs: a 0.25 magnitude error in the distance modulus translates to a 22% error in the derived cluster age [18]. Stellar mass loss is a recent idea for lowering the GC t_0[19], but observations constrain the reduction in t_0 to be less than ~ 1 Gyr. Allowing $\sim 1 - 2$ Gyr for galaxy and GC formation, we conclude that $t_0 \gtrsim 11$ Gyr from GCs, with $t_0 \approx 13$ Gyr a "likely" lower limit on t_0, obtained by pushing many but not all the parameters to their limits.

The GC age estimates are of course based on standard stellar evolution calculations. But the solar neutrino problem reminds us that we are not really sure that we understand how even our nearest star operates; and the sun plays an important role in calibrating stellar evolution, since it is the only star whose age we know independently (from radioactive dating of early solar system material). What if the GC age estimates are wrong for some unknown reason?

The only independent estimates of the age of the universe come from cosmochronometry — the chemical evolution of the Galaxy — and white dwarf cooling. Cosmochronometry age estimates are sensitive to a number of uncertain effects such as the formation history of the disk and its stars, and possible actinide destruction in stars [20]. Age estimates also come from the cooling of white dwarfs in the neighborhood of the sun. The key observation is that there is a lower limit to the luminosity and therefore also the temperature of nearby white dwarfs; although dimmer ones could have been seen, none have been found. The only plausible explanation is that the white dwarfs have not had sufficient time to cool to lower temperatures, which initially led to an estimate of 9.3 ± 2 Gyr for the age of the Galactic disk [21]. Since there is evidence that the stellar disk of our Galaxy is about 2 Gyr younger than the oldest GCs [22], this in turn gave an estimate of the age of the universe of $t_0 \sim 11 \pm 2$ Gyr. More recent analyses [23] conclude that sensitivity to disk star formation history, and to effects on the white dwarf cooling rates due to C/O separation at crystallization and possible presence of trace elements such as ^{22}Ne, allow a rather wide range of ages for the disk of about 10 ± 4 Gyr. The latest determination of the white dwarf luminosity function, using white dwarfs in proper motion binaries, leads to a somewhat lower minimum luminosity and therefore a somewhat higher estimate of the age of the disk of $\sim 10.5^{+2.5}_{-1.5}$ Gyr; it follows that $t_0 \geq 11.5$ Gyr [24].

Suppose that the GC stellar age estimates that $t_0 \gtrsim 13$ Gyr are right. Fig. 1 shows that $t_0 > 13$ Gyr implies that $H_0 \leq 50$ km s^{-1} Mpc^{-1} for $\Omega = 1$, and that $H_0 \leq 81$ km s^{-1} Mpc^{-1} even for Ω_0 as small as 0.2 (in flat cosmologies with $\Omega_0 + \Omega_\Lambda = 1$).

4 Hubble Parameter H_0

The Hubble parameter $H_0 \equiv 100h$ km s^{-1} Mpc^{-1} remains uncertain by about a factor of two: $0.4 \lesssim h \lesssim 1$. Sandage has long contended that $h \approx 0.5$, and he still concludes [25,26] that the latest data are consistent with $h = 0.55 \pm 0.1$. de Vaucouleurs long
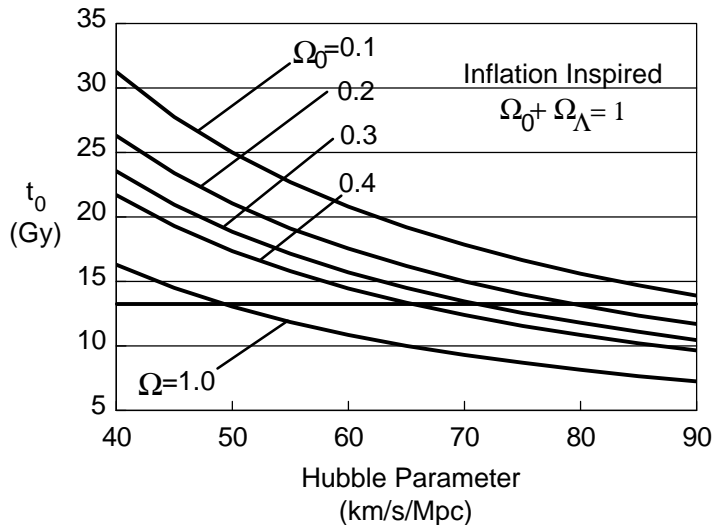
Figure 1. Age of the universe t_0 as a function of Hubble parameter H_0 in inflation inspired models with $\Omega_0 + \Omega_\Lambda = 1$, for several values of the present-epoch cosmological density parameter Ω_0.

contended that $h \approx 1$. A majority of observers currently favor a value intermediate between these two extremes (recent reviews include [27,28,29,30]).

The Hubble parameter has been measured in two basic ways: (A) Measuring the distance to some nearby galaxies, typically by measuring the periods and luminosities of Cepheid variables in them; and then using these "calibrator galaxies" to set the zero point in any of the several methods of measuring the relative distances to galaxies. (B) Using fundamental physics to measure the distance to some distant object directly, thereby avoiding at least some of the uncertainties of the cosmic distance ladder [31]. The difficulty with method (A) is that there are so far only a handful of calibrator galaxies close enough for Cepheids to be resolved in them. However, the success of the HST Cepheid measurement of the distance to M100 [2] shows that the HST Key Project on the Extragalactic Distance Scale can significantly increase the set of calibrator galaxies within a few years. Adaptive optics from the ground may also be able to contribute to this effort, although I am not very impressed by the first published result of this approach [32]. The difficulty with method (B) is that in every case studied so far, some aspect of the observed system or the underlying physics remains somewhat uncertain. It is nevertheless remarkable that the results of several different methods of type (B) are rather similar, and indeed not very far from those of method (A). This gives reason to hope for convergence.

4.1 (A) Relative Distance Methods

One piece of good news is that the several methods of measuring the relative distances to galaxies now mostly seem to be consistent with each other [28,29]. These methods use either (1) "standard candles" or (2) empirical relations between two measurable properties of a galaxy, one distance-independent and the other distance-dependent. The old favorite standard candle is Type Ia supernovae; a new one is the apparent maximum luminosity of planetary nebulae [28]. Sandage [26] and others [33] get low values of $h \approx 0.55$ from HST Cepheid distances to SN Ia host galaxies, including the seven SNe Ia with well-observed maxima that lie in six galaxies with HST Cepheid distances. There are claims that taking account of an empirical relationship between the SN Ia light curve shape and maximum luminosity leads to higher h [34], but Sandage and Tammann counter that any such effect is small [35,26] since they have not used those "abnormal" SNe Ia that would be significantly affected. The old favorite empirical relation used as a relative distance indicator is the Tully-Fisher relation between the rotation velocity and luminosity of spiral galaxies (and the related Faber-Jackson or $D_n - \sigma$ relation); a newer one is based on the decrease in the fluctuations in elliptical galaxy surface brightness on a given angular scale as galaxies are seen at greater distances [36].

4.2 (B) Fundamental Physics Approaches

The fundamental physics approaches involve either Type Ia or Type II supernovae, the Sunyaev-Zel'dovich (S-Z) effect, or gravitational lensing.

The ^{56}Ni radioactivity method for determining H_0 using Type Ia SN avoids the uncertainties of the distance ladder by calculating the absolute luminosity of Type Ia supernovae from first principles using a plausible but as yet unproved physical model. The first result obtained was that $h = 61 \pm 10$ [37]; however, another study [38] found that uncertainties in extinction (i.e., light absorption) toward each supernova increases the range of allowed h. Demanding that the ^{56}Ni radioactivity method agree with an expanding photosphere approach leads to $H_0 = 60^{+14}_{-11}$ [39]. The expanding photosphere method compares the expansion rate of the SN envelope measured by redshift with its size increase inferred from its temperature and magnitude. This approach was first applied to Type II SN; the 1992 result $h = 0.6 \pm 0.1$ [40] was subsequently revised upward by the same authors to $h = 0.73 \pm 0.06 \pm 0.07$ [41]. However, there are various complications with the physics of the expanding envelope [42].

The S-Z effect is the Compton scattering of microwave background photons from the hot electrons in a foreground galaxy cluster. This can be used to measure H_0 since properties of the cluster gas measured via the S-Z effect and from X-ray observations have different dependences on H_0. The result from the first cluster for which sufficiently detailed data was available, A665 (at $z = 0.182$), was $h = (0.4-0.5) \pm 0.12$ [43]; combining this with data on A2218 ($z = 0.171$) raises this somewhat to $h = 0.55 \pm 0.17$ [44]. Early results from the ASCA X-ray satellite gave $h = 0.47 \pm 0.17$ for A665 ($z = 0.182$) and

$h = 0.41^{+0.15}_{-0.12}$ for CL0016+16 ($z = 0.545$) [45]. A few S-Z results have been obtained using millimeter-wave observations, and this promising method should allow many more such measurements soon [46]. Corrections for the near-relativistic electron motions will raise these estimates for H_0 a little [47], but it seems clear that the S-Z results favor a smaller value than many optical astronomers obtain. However, since the S-Z measurement of H_0 is affected by the orientation of the cluster ellipticity with respect to the line of sight, this will only become convincing with observations of a significant number of additional clusters. Fortunately, this now appears to be possible within the next several years.

Several quasars have been observed to have multiple images separated by a few arc seconds; this phenomenon is interpreted as arising from gravitational lensing of the source quasar by a galaxy along the line of sight. In the first such system discovered, QSO 0957+561 ($z = 1.41$), the time delay Δt between arrival at the earth of variations in the quasar's luminosity in the two images has been measured to be 409 ± 23 days [48], although other authors found a value of 540 ± 12 days [49]. The shorter Δt has now been confirmed by the observation of a sharp drop in Image A of about 0.1 mag in late December 1994 [50], followed by a similar drop in Image B about 405-420 days later (R. Schild and E.L. Turner, private communications). Since $\Delta t \approx \theta^2 H_0^{-1}$, this observation allows an estimate of the Hubble parameter, with the results $h = 0.50 \pm 0.17$ [51], or $h = 0.63 \pm 0.21$ ($h = 0.42 \pm 0.14$) including (neglecting) dark matter in the lensing galaxy [52], with additional uncertainties associated with possible microlensing and unknown matter distribution in the lensing galaxy. However, recent deep images have allowed mapping of the gravitational potential of the cluster (at $z = 0.36$) in which the lensing galaxy lies using weak gravitational lensing, which leads to the conclusion that $h \leq 0.70(1.1\text{yr}/\Delta t)$ [53]. Also, detailed study of the lensed QSO images (which include a jet) constrains the lensing and implies $h = 0.82(1 - \kappa)(1.1\text{yr}/\Delta t) < 0.82$, where the upper limit follows because the convergence due to the cluster $\kappa > 0$, or alternatively $h = 0.82(\sigma/322\,\text{km}\,\text{s}^{-1})^2(1.1\text{yr}/\Delta t)$ without uncertainty concerning the cluster if the one-dimensional velocity dispersion σ in the core of the giant elliptical galaxy responsible for the lensing can be measured [54]. Although the uncertainty in H_0 remains rather large, it is reassuring that this method gives results consistent with the other determinations. The time-delay method is promising, and when delays are reliably measured in several other multiple-image quasar systems, that should lead to a reliable value for h.

4.3 Correcting for Virgocentric Infall

What about the recent HST Cepheid measurement of H_0, giving $h \approx 0.8$ [2]? This calculated value is based on neither of the two methods (A) or (B) above, and I do not regard it as being very reliable. Instead this result is obtained by assuming that M100 is at the core of the Virgo cluster, and dividing the sum of the recession velocity of Virgo, about 1100 km s^{-1}, plus the calculated "infall velocity" of the local group toward Virgo, about 300 km s^{-1}, by the measured distance to M100 of 17.1 Mpc. (These recession and infall velocities are both a little on the high side, compared to other values one finds in the

literature.) Adding the "infall velocity" is necessary in this method in order to correct the Virgo recession velocity to what it would be were it not for the gravitational attraction of Virgo for the Local Group of galaxies, but the problem with this is that the net motion of the Local Group with respect to Virgo is undoubtedly affected by much besides the Virgo cluster — e.g., the "Great Attractor." For example, in our CHDM supercomputer simulations (which appear to be a rather realistic match to observations) Anatoly Klypin and I have found that galaxies and groups at about 20 Mpc from a Virgo-sized cluster often have net outflowing rather than infalling velocities. Note that if the net "infall" of M100 were smaller, or if M100 were in the foreground of the Virgo cluster (in which case the actual distance to Virgo would be larger than 17.1 Mpc), then the indicated H_0 would be smaller.

The authors of Ref. [2] gave an alternative argument that avoids the "infall velocity" uncertainty: the relative galaxy luminosities indicate that the Coma cluster is about six times farther away than the Virgo cluster, and peculiar motions of the Local Group and the Coma cluster are relatively small corrections to the much larger recession velocity of Coma; dividing the recession velocity of the Coma cluster by six times the distance to M100 again gives $H_0 \approx 80$. However, this approach still assumes that M100 is in the core rather than the foreground of the Virgo cluster; and in deducing the relative distance of the Coma and Virgo clusters it assumes that the galaxy luminosity functions in each are comparable, which is uncertain in view of the very different environments. More general arguments by the same authors [55] lead them to conclude that $H_0 = 73 \pm 11$ regardless of where M100 lies in the Virgo cluster. But Tammann et al. [26], using all the available HST Cepheid distances and their own complete sample of Virgo spirals, conclude that $H_0 \approx 54$.

To summarize, many observers, using mainly method (A), favor a value $h \approx 0.6 - 0.8$ although Sandage's group and some others continue to get $h \approx 0.5 - 0.6$, while the methods I have grouped together as (B) typically lead to $h \approx 0.4 - 0.7$. The fact that the latter measurements are mostly of more distant objects has suggested [56] that the local universe may actually be underdense and therefore be expanding faster than is typical. But in reasonable models where structure forms from Gaussian fluctuations via gravitational instability, it is extremely unlikely that a sufficiently large region has a density sufficiently smaller than average to make more than a rather small difference in the measured value of h [57]. Moreover, the small dispersion in the corrected maximum luminosity of distant SNe Ia found by the LBL Supernova Cosmology Project [58] compared to nearby SNe Ia shows directly that the local and cosmological values of H_0 are approximately equal.

There has been recent observational progress in both methods (A) and (B), and I think it likely that the Hubble parameter will be known reliably to 10% within a few years. But at present the uncertainty remains rather large $0.4 < h < 0.8$, and we must keep an open mind.

5 Cosmological Constant Λ, and t_0 Again

Inflation is the only known solution to the horizon and flatness problems and the avoidance of too many GUT monopoles. And inflation has the added bonus that with no extra charge (except the perhaps implausibly fine-tuned adjustment of the self-coupling of the inflaton field to be adequately small), simple inflationary models predict a near-Zel'dovich spectrum (i.e., with $n_p \approx 1$) of adiabatic Gaussian primordial fluctuations — which seems to be consistent with observations. All simple inflationary models predict that the curvature constant k is vanishingly small, although inflationary models that are extremely contrived (at least, to my mind) can be constructed with negative curvature and therefore $\Omega_0 \lesssim 1$ without a cosmological constant [59]. Thus most authors who consider inflationary models impose the condition $k = 0$, or $\Omega_0 + \Omega_\Lambda = 1$ where $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$. This is what is assumed in ΛCDM models, and it is what was assumed in Fig. 1. (I hope it has been clear from the foregoing that I use Ω to refer only to the density of matter and energy, not including the cosmological constant, whose contribution in the Ω units is Ω_Λ.)

I know of no one (except possibly Lev Kofman) who actually finds the idea of a nonvanishing Λ intrinsically attractive. There is no known physical reason why Λ should be so small (from the viewpoint of particle physics), though there is also no known reason why it should vanish. The most unattractive features of $\Lambda \neq 0$ cosmologies are the fact that Λ must become important only at relatively low redshift — why not much earlier or much later? — and also that $\Omega_\Lambda \gtrsim \Omega_0$ implies that the universe has recently entered an inflationary epoch (with a de Sitter horizon comparable to the present horizon). The main motivations for $\Lambda > 0$ cosmologies are (1) reconciling inflation with observations that seem to imply $\Omega_0 \lesssim 1$, and (2) avoiding a contradiction between the lower limit $t_0 \gtrsim 13$ Gyr from globular clusters and $t_0 = (2/3)H_0^{-1} = 6.52h^{-1}$ Gyr for the standard $\Omega = 1$, $\Lambda = 0$ Einstein-de Sitter cosmology, if it is really true that $h > 0.5$.

The cosmological effects of a cosmological constant are not difficult to understand [60,61]. With a positive Λ, there is a repulsion of space by space. In the early universe, the density of energy and matter is far more important than Λ on the r.h.s. of the Friedmann equation. But the average matter density decreases as the universe expands, and at a rather low redshift $z \sim 0.3$ the Λ term finally becomes dominant. If it has been adjusted just right, Λ can almost balance the attraction of the matter, and the expansion nearly stops: for a long time, the scale factor $a \equiv (1+z)^{-1}$ increases very slowly, although it ultimately starts increasing exponentially as the universe starts inflating under the influence of the increasingly dominant Λ term. The existence of a period during which expansion slows while the clock runs explains why t_0 can be greater than for $\Lambda = 0$, but this also shows that there is a increased likelihood of finding galaxies at the redshift interval when the expansion slowed, and a correspondingly increased opportunity for lensing of quasars (which mostly lie at higher redshift $z \gtrsim 2$) by these galaxies.

The frequency of such lensed quasars is about what would be expected in a standard

$\Omega = 1$, $\Lambda = 0$ cosmology, so this data sets fairly stringent upper limits: $\Omega_\Lambda \leq 0.70$ at 90% C.L. [62,63], with more recent data giving even tighter constraints: $\Omega_\Lambda < 0.66$ at 95% confidence [64]. This limit could perhaps be weakened if there were (a) significant extinction by dust in the E/S0 galaxies responsible for the lensing or (b) rapid evolution of these galaxies, but there is much evidence that these galaxies have little dust and have evolved only passively for $z \lesssim 1$ [65].

A weaker but independent constraint comes from the cosmic background radiation data. In standard $\Omega = 1$ models, the quantity $\ell(\ell + 1)C_\ell$ (where $C_\ell =< a^2_{\ell,m} >_m$ is the average of squared coefficients of the spherical harmonic expansion of the CMB data) is predicted to be roughly constant for $2 \leq \ell \lesssim 10$ (with an increase for higher multiples toward the Doppler peak at $\ell \sim 200$), while in models with $\Lambda > 0$ $\ell(\ell + 1)C_\ell$ is predicted to dip before rising toward the Doppler peak. Comparison with the two-year COBE data, in which such a dip is not seen, implies that $\Omega_\Lambda \leq 0.78$ at the 90% C.L. [66].

Yet another constraint comes from number counts of bright E/S0 galaxies in HST images [67], since these galaxies appear to have evolved rather little since $z \sim 1$ [65]. The number counts are just as expected in the $\Omega = 1$, $\Lambda = 0$ Einstein-de Sitter cosmology. Even allowing for uncertainties due to evolution and merging of these galaxies, this data would allow Ω_Λ as large as 0.8 in flat cosmologies only in the unlikely event that half the Sa galaxies in the deep HST images were misclassified as E/S0. This approach may be very promising for the future, as the available deep HST image data and our understanding of galaxy evolution both increase.

A model-dependent constraint comes from a detailed simulation of ΛCDM [68]: a COBE-normalized model with $\Omega_0 = 0.3$ and $h = 0.7$ has far too much power on small scales to be consistent with observations, even allowing for seemingly unrealistic antibiasing of galaxies with respect to dark matter. For ΛCDM models, the only solution appears to be raising Ω_0, lowering H_0, and tilting the spectrum ($n_p < 1$) [68], though of course one could alternatively modify the primordial power spectrum in other ways.

Fig. 1 shows that with $\Omega_\Lambda \leq 0.7$, the cosmological constant does not lead to a very large increase in t_0 compared to the Einstein-de Sitter case, although it may still be enough to be significant. For example, the constraint that $t_0 \geq 13$ Gyr requires $h \leq 0.5$ for $\Omega = 1$ and $\Lambda = 0$, but this becomes $h \leq 0.70$ for flat cosmologies with $\Omega_\Lambda \leq 0.66$.

6   Measuring Ω_0

6.1   Very Large Scale Measurements

Although it would be desirable to measure Ω_0 and Λ through their effects on the large-scale geometry of space-time, this has proved difficult in practice since it requires comparing objects at higher and lower redshift, and it is hard to separate the effects of the evolution of the objects from those of the evolution of the universe. For example, in "redshift-volume" tests involving number counts of galaxies per redshift interval, how

can we tell whether the galaxies at redshift $z \sim 1$ correspond to those at $z \sim 0$? Several galaxies at higher redshift might have merged, and galaxies might have formed or changed luminosity at lower redshift. Eventually, with extensive surveys of galaxy properties as a function of redshift using the largest telescopes such as Keck, it should be possible to perform these classical cosmological tests at least on a particular class of galaxies — that is one of the goals of the Keck DEEP project.

At present, perhaps the most promising technique involves searching for Type Ia supernovae (SNe Ia) at high-redshift, since these are the brightest supernovae and the spread in their intrinsic brightness appears to be relatively small. Saul Perlmutter, Gerson Goldhaber, and collaborators have recently demonstrated the feasibility of finding significant numbers of such supernovae [69], but a dedicated campaign of follow-up observations of each one will be required in order to measure Ω_0 by determining how the apparent brightness of the supernovae depends on their redshift. This is therefore a demanding project. It initially appeared that ~ 100 high redshift SNe Ia would be required to achieve a 10% measurement of $q_0 = \Omega_0/2 - \Omega_\Lambda$. However, using the correlation mentioned earlier between the absolute luminosity of a SN Ia and the shape of its light curve (e.g., slower decline correlates with higher peak luminosity), it now appears possible to reduce the number of SN Ia required. The Perlmutter group has now analyzed seven high redshift SN Ia by this method, with a tentative result that $q_0 \sim 0.5$ and in any case $q_0 \gtrsim 0$ [70]. In November 1995 they discovered an additional 11 high-redshift SN Ia, and they have just discovered an additional seven. Preliminary results from the next 11 SN Ia are consistent with $q_0 \sim 0.5$, and there are no peculiar or anomalous ones in the dataset (Perlmutter, private communication). Two other groups, collaborations from ESO and MSSSO/CfA/CTIO, are also searching for high-redshift supernovae to measure q_0, and have found at least a few. There has also been recent progress understanding the physical origin of the SN Ia luminosity-light curve correlation. At the present rate of progress, a reliable answer may be available within perhaps a year or so if a consensus emerges from these efforts.

6.2 Large-scale Measurements

Ω_0 has been measured with some precision on a scale of about $\sim 50\,h^{-1}\,\mathrm{Mpc}$, using the data on peculiar velocities of galaxies, and on a somewhat larger scale using redshift surveys based on the IRAS galaxy catalog. Since the results of all such measurements to date have recently been reviewed in detail [71], I will only comment briefly on them. The analyses such as "POTENT" that try to recover the scalar velocity potential from the galaxy peculiar velocities are looking increasingly reliable, since they reproduce the observed large scale distribution of galaxies – that is, many galaxies are found where the converging velocities indicate that there is a lot of matter, and there are voids in the galaxy distribution where the diverging velocities indicate that the density is lower than average. The comparison of the IRAS redshift surveys with POTENT and related analyses typically give fairly large values for the parameter $\beta_I \equiv \Omega_0^{0.6}/b_I$ (where b_I is the biasing parameter for IRAS galaxies), corresponding to $0.3 \lesssim \Omega_0 \lesssim 3$ (for an assumed $b_I = 1.15$). It is not

clear whether it will be possible to reduce the spread in these values significantly in the near future — probably both additional data and a better understanding of systematic and statistical effects will be required.

A particularly simple way to deduce a lower limit on Ω_0 from the POTENT peculiar velocity data has recently been proposed [72], based on the fact that high-velocity outflows from voids are not expected in low-Ω models. Data on just one void indicates that $\Omega_0 \geq 0.3$ at the 97% C.L. This argument is independent of assumptions about Λ or galaxy formation, but of course it does depend on the success of POTENT in recovering the peculiar velocities of galaxies.

However, for the particular cosmological models that I am focusing on in this review — CHDM and ΛCDM — stronger constraints are available. This is because these models, in common with almost all CDM variants, assume that the probability distribution function (PDF) of the primordial fluctuations was Gaussian. Evolution from a Gaussian initial PDF to the non-Gaussian mass distribution observed today requires considerable gravitational nonlinearity, i.e. large Ω. The PDF deduced by POTENT from observed velocities (i.e., the PDF of the mass, if the POTENT reconstruction is reliable) is far from Gaussian today, with a long positive-fluctuation tail. It agrees with a Gaussian initial PDF if and only if Ω is about unity or larger: $\Omega_0 < 1$ is rejected at the 2σ level, and $\Omega_0 \leq 0.3$ is ruled out at $\geq 4\sigma$ [73].

6.3 Measurements on Scales of a Few Mpc

On smaller length scales, there are many measurements that are consistent with a smaller value of Ω_0 [74]. For example, the cosmic virial theorem gives $\Omega(\sim 1h^{-1}\,\mathrm{Mpc}) \approx 0.15[\sigma(1h^{-1}\,\mathrm{Mpc})/(300\,\mathrm{km\,s^{-1}})]^2$, where $\sigma(1h^{-1}\,\mathrm{Mpc})$ here represents the relative velocity dispersion of galaxy pairs at a separation of $1h^{-1}\,\mathrm{Mpc}$. Although the classic paper [75] which first measured $\sigma(1h^{-1}\,\mathrm{Mpc})$ using a large redshift survey (CfA1) got a value of 340 km s^{-1}, this result is now known to be in error since the entire core of the Virgo cluster was inadvertently omitted [76]; if Virgo is included, the result is $\sim 500 - 600$ km s^{-1} [77,76], corresponding to $\Omega(\sim 1h^{-1}\,\mathrm{Mpc}) \approx 0.4 - 0.6$. Various redshift surveys give a wide range of values for $\sigma(1h^{-1}\,\mathrm{Mpc}) \sim 300 - 750$ km s^{-1}, with the most salient feature being the presence or absence of rich clusters of galaxies; for example, the IRAS galaxies, which are not found in clusters, have $\sigma(1h^{-1}\,\mathrm{Mpc}) \approx 320$ km s^{-1} [78], while the northern CfA2 sample, with several rich clusters, has much larger σ than the SSRS2 sample, with only a few relatively poor clusters [79,76,80]. It is evident that the $\sigma(1h^{-1}\,\mathrm{Mpc})$ statistic is not a very robust one.

A standard method for estimating Ω on scales of a few Mpc is based on applying virial estimates to groups and clusters of galaxies to try to deduce the total mass of the galaxies including their dark matter halos from the velocities and radii of the groups; roughly, $GM \sim rv^2$. (What one actually does is to assume that all galaxies have the same mass-to-light ratio M/L, given by the median M/L of the groups, and integrate over the luminosity function to get the mass density [81,82,83].) The typical result is

that $\Omega(\sim 1\,h^{-1}\,\mathrm{Mpc}) \sim 0.1 - 0.2$. However, such estimates are at best lower limits, since they can only include the mass within the region where the galaxies in each group can act as test particles. In CHDM simulations, my colleagues and I [84] have found that the effective radius of the dark matter distribution associated with galaxy groups is typically 2-3 times larger than that of the galaxy distribution. Moreover, we find a velocity biasing [85] factor in CHDM groups $b_v^{grp} \equiv v_{\mathrm{gal,rms}}/v_{\mathrm{DM,rms}} \approx 0.75$, whose inverse squared enters in the Ω estimate. Finally, we find that groups and clusters are typically elongated, so only part of the mass is included in spherical estimators. These factors explain how it can be that our $\Omega = 1$ CHDM simulations produce group velocities that are fully consistent with those of observed groups, even with statistical tests such as the median rms group velocity vs. the fraction of galaxies grouped [86,84]. This emphasizes the point that local estimates of Ω are at best lower limits on its true value.

Another approach to estimating Ω from information on relatively small scales has been pioneered by Peebles [87]. It is based on using the least action principle (LAP) to reconstruct the trajectories of the Local Group galaxies, and the assumption that the mass is concentrated around the galaxies. This is a reasonable assumption in a low-Ω universe, but it is not at all what must occur in an $\Omega = 1$ universe where most of the mass must lie between the galaxies. Although comparison with $\Omega = 1$ N-body simulations showed that the LAP often succeeds in qualitatively reconstructing the trajectories, the mass is systematically underestimated by a large factor by the LAP method [88]. Unexpectedly, a different study [89] found that the LAP method underestimates Ω by a factor of 4-5 even in an $\Omega_0 = 0.2$ simulation; the authors say that this discrepancy is due to the LAP neglecting the effect of "orphans" — dark matter particles that are not members of any halo. Shaya, Peebles, and Tully [90] have recently attempted to apply the LAP to galaxies in the local supercluster, again getting low Ω_0. The LAP approach should be more reliable on this larger scale, but the method still must be calibrated on N-body simulations of both high- and low-Ω_0 models before its biases can be quantified.

6.4 Estimates on Galaxy Halo Scales

Recent work by Zaritsky and White [91] and collaborators has shown that spiral galaxies have massive halos. A classic paper by Little and Tremaine [97] argued that the available data on the Milky Way satellite galaxies required that the Galaxy's halo terminate at about 50 kpc, with a total mass of only about $2.5 \times 10^{11} M_\odot$. But by 1991, new data on local satellite galaxies, especially Leo I, became available, and the Little-Tremaine estimator increased to $1.25 \times 10^{12} M_\odot$. A recent, detailed study finds a mass inside 50 kpc of $(5.4 \pm 1.3) \times 10^{11} M_\odot$ [98]. Zaritsky and collaborators have collected data on satellites of isolated spiral galaxies, and conclude that the fact that the relative velocities do not fall off out to a separation of at least 200 kpc shows that massive halos are the norm. The typical rotation velocity of $\sim 200 - 250$ km s^{-1} implies a mass within 200 kpc of $2 \times 10^{12} M_\odot$. A careful analysis taking into account selection effects and satellite orbit uncertainties concluded that the indicated value of Ω_0 exceeds 0.13 at 90% confidence,

14

with preferred values exceeding 0.3 [91]. Newer data suggesting that relative velocities do not fall off out to a separation of at least 300 kpc will raise these Ω_0 estimates [92].

However, if galaxy dark matter halos are really so extended and massive, that would imply that when such galaxies collide, the resulting tidal tails of debris cannot be flung very far. Therefore, the observed merging galaxies with extended tidal tails such as NGC 4038/39 (the Antennae) and NGC 7252 probably have halo:(disk+bulge) mass ratios less than 10:1 [93], unless the stellar tails are perhaps made during the collision process from gas that was initially far from the central galaxies (J. Ostriker, private communication, 1996); the latter possibility can be checked by determining the ages of the stars in these tails.

A direct way of measuring the mass and spatial extent of many galaxy dark matter halos is to look for the small distortions of distant galaxy images due to gravitational lensing by foreground galaxies. This technique was pioneered by Tyson et al. [94]. Though the results were inconclusive, powerful constraints could perhaps be obtained from deep HST images or ground-based images with excellent seeing. Such fields would also be useful for measuring the correlated distortions of galaxy images from large-scale structure by weak gravitational lensing; although a pilot project [95] detected only a marginal signal, a reanalysis detected a significant signal suggesting that $\Omega_0 \sigma_8 \sim 1$ [96]. Several groups are planning major projects of this sort.

7 Clusters

7.1 Cluster Baryons vs. Big Bang Nucleosynthesis

A recent review [9] of Big Bang Nucleosynthesis (BBN) and observations indicating primordial abundances of the light isotopes concludes that $0.009h^{-2} \leq \Omega_b \leq 0.02h^{-2}$ for concordance with all the abundances, and $0.006h^{-2} \leq \Omega_b \leq 0.03h^{-2}$ if only deuterium is used. For $h = 0.5$, the corresponding upper limits on Ω_b are 0.08 and 0.12, respectively. The recent observations [99] of a possible deuterium line in a hydrogen cloud at redshift $z = 3.32$ indicating a deuterium abundance of $\sim 2 \times 10^{-4}$ (and therefore $\Omega_b \leq 0.006h^{-2}$) are contradicted by similar observations by Tytler and collaborators [100] in systems at $z = 3.57$ and $z = 2.504$ but with a deuterium abundance about ten times lower, consistent with solar system measurements of D and ^{3}He and implying $\Omega_b h^2 = 0.024 \pm 0.05$, or Ω_b in the range 0.08-0.11 for $h = 0.5$. (If these represent the true D/H, then the earlier observations [99] were most probably of a Lyα forest line. However, Rugers and Hogan [101] argue that the width of their $z = 3.32$ absorption features is better fit by deuterium, although they admit that only a statistical sample of absorbers will settle the issue. There is a new possible detection of D at $z = 4.672$ in the absorption spectrum of QSO BR1202-0725 [102] and at $Z = 3.086$ toward Q 0420-388 [103]. But Tytler [100] argues that the two systems he and his colleagues have analyzed are much more convincing as real

detections of deuterium, and it is surely significant that they measure the same D/H in both systems.)

White et al. [104] have emphasized that recent X-ray observations of clusters, especially Coma, show that the abundance of baryons, mostly in the form of gas (which typically amounts to several times the total mass of the cluster galaxies), is about 20% if h is as low as 0.5. For the Coma cluster they find that the baryon fraction within the Abell radius is

$$f_b \equiv \frac{M_b}{M_{tot}} \geq 0.009 + 0.050h^{-3/2},$$

where the first term comes from the galaxies and the second from gas. If clusters are a fair sample of both baryons and dark matter, as they are expected to be based on simulations, then this is 2-3 times the amount of baryonic mass expected on the basis of BBN in an $\Omega = 1$, $h \approx 0.5$ universe, though it is just what one would expect in a universe with $\Omega_0 \approx 0.3$. The fair sample hypothesis implies that

$$\Omega_0 = \frac{\Omega_b}{f_b} = 0.3 \left(\frac{\Omega_b}{0.06} \right) \left(\frac{0.2}{f_b} \right).$$

A recent review of gas in a sample of clusters [105] finds that the baryon mass fraction within about 1 Mpc lies between 10 and 22% (for $h = 0.5$; the limits scale as $h^{-3/2}$), and argues that it is unlikely that (a) the gas could be clumped enough to lead to significant overestimates of the total gas mass — the main escape route considered in [104] (cf. also [106]). The gas mass would also be overestimated if large tangled magnetic fields provide a significant part of the pressure in the central regions of some clusters [107]; this can be checked by observation of Faraday rotation of sources behind clusters [108]. If $\Omega = 1$, the alternatives are then either (b) that clusters have more mass than virial estimates based on the cluster galaxy velocities or estimates based on hydrostatic equilibrium [109] of the gas at the measured X-ray temperature (which is surprising since they agree [110]), or (c) that the BBN upper limit on Ω_b is wrong. It is interesting that there are indications from weak lensing [111] that at least some clusters may actually have extended halos of dark matter — something that is expected to a greater extent if the dark matter is a mixture of cold and hot components, since the hot component clusters less than the cold [84,112,113]. If so, the number density of clusters as a function of mass is higher than usually estimated, which has interesting cosmological implications (e.g. σ_8 is higher than usually estimated). It is of course possible that the solution is some combination of alternatives (a), (b), and (c). If none of the alternatives is right, then the only conclusion left is that $\Omega_0 \approx 0.3$. The cluster baryon problem is clearly an issue that deserves very careful examination.

7.2 Cluster Morphology

Richstone, Loeb, and Turner [114] showed that clusters are expected to be evolved — i.e. rather spherical and featureless — in low-Ω cosmologies, in which structures form at

relatively high redshift, and that clusters should be more irregular in $\Omega = 1$ cosmologies, where they have formed relatively recently and are still undergoing significant merger activity. There are very few known clusters that seem to be highly evolved and relaxed, and many which are irregular — some of which are obviously undergoing mergers now or have recently done so (see e.g. [115]). This disfavors low-Ω models, but it remains to be seen just how low. Recent papers have addressed this. In one [116] a total of 24 CDM simulations with $\Omega = 1$ or 0.2, the latter with $\Omega_\Lambda = 0$ or 0.8, were compared with data on a sample of 57 clusters. The conclusion was that clusters with the observed range of X-ray morphologies are very unlikely in the low-Ω cosmologies. However, these simulations have been criticized because the $\Omega_0 = 0.2$ ones included rather a large amount of ordinary matter: $\Omega_b = 0.1$. (This is unrealistic both because $h \approx 0.8$ provides the best fit for $\Omega_0 = 0.2$ CDM, but then the standard BBN upper limit is $\Omega_b < 0.02h^{-2} = 0.03$; and also because observed clusters have a gas fraction of $\sim 0.15(h/0.5)^{-3/2}$.) Another study [117] using dissipationless simulations and not comparing directly to observational data found that ΛCDM with $\Omega_0 = 0.3$ and $h = 0.75$ produced clusters with some substructure, perhaps enough to be observationally acceptable. Clearly, this important issue deserves study with higher resolution hydrodynamic simulations, with a range of assumed Ω_b, and possibly including at least some of the additional physics associated with the galaxies which must produce the metallicity observed in clusters, and perhaps some of the heat as well. Better statistics for comparing simulations to data may also be useful [118].

7.3   Cluster Evolution

There is evidence for strong evolution of clusters at relatively low redshift, both in their X-ray properties [119] and in the properties of their galaxies. In particular, there is a strong increase in the fraction of blue galaxies with increasing redshift (the "Butcher-Oemler effect"), which may be difficult to explain in a low-density universe [121]. Field galaxies do not appear to show such strong evolution; indeed, a recent study concludes that over the redshift range $0.2 \leq z \leq 1.0$ there is no significant evolution in the number density of "normal" galaxies [120]. This is compatible with the predictions of CHDM with two neutrinos sharing a total mass of about 5 eV [122] (see below).

8   Early Structure Formation

In linear theory, adiabatic density fluctuations grow linearly with the scale factor in an $\Omega = 1$ universe, but more slowly if $\Omega < 1$ with or without a cosmological constant [74]. As a result, if fluctuations of a certain size in an $\Omega = 1$ and an $\Omega_0 = 0.3$ theory are equal in amplitude at the present epoch ($z = 0$), then at higher redshift the fluctuations in the low-Ω model had higher amplitude. Thus, structures typically form earlier in low-Ω models than in $\Omega = 1$ models.

Since quasars are seen at the highest redshifts, they have been used to try to constrain $\Omega = 1$ theories, especially CHDM which because of the hot component has additional suppression of small-scale fluctuations that are presumably required to make early structure (e.g., [123]). The difficulty is that dissipationless simulations predict the number density of halos of a given mass as a function of redshift, but not enough is known about the nature of quasars — for example, the mass of the host galaxy — to allow a simple prediction of the number of quasars as a function of redshift in any given cosmological model. A recent study [124] concludes that very efficient cooling of the gas in early structures, and angular momentum transfer from it to the dark halo, allows for formation of *at least* the observed number of quasars even in models where most galaxy formation occurs late.

Observers are now beginning to see significant numbers of what appear to be the central regions of galaxies in an early stage of their formation at redshifts $z = 3 - 3.5$ [125] — although, as with quasars, a danger in using systems observed by emission is that they may not be typical. As additional observations clarify the nature of these objects, they can perhaps be used to constrain cosmological parameters and models.

Another sort of high redshift object which may hold more promise for constraining theories is damped Lyman α systems (DLAS). DLAS are dense clouds of neutral hydrogen, generally thought to be protogalactic disks, which are observed as wide absorption features in quasar spectra [126]. They are relatively common, seen in roughly a third of all quasar spectra, so statistical inferences about DLAS are possible. At the highest redshift for which data is published, $z = 3 - 3.4$, the density of neutral gas in such systems in units of critical density is $\Omega_{gas} \approx 0.6\%$, comparable to the total density of visible matter in the universe today [127]. Several recent papers [128] pointed out that the CHDM model with $\Omega_\nu = 0.3$ could not produce such a high Ω_{gas}. However, my colleagues and I showed that CHDM with $\Omega_\nu = 0.2$ could do so [129], since the power spectrum on small scales is a very sensitive function of the total neutrino mass in CHDM models. This theory makes two crucial predictions [129]: Ω_{gas} must fall off at higher redshifts z, and the DLAS at $z \gtrsim 3$ correspond to systems of internal rotation velocity or velocity dispersion less than about 100 km s^{-1} (this can be measured from the Doppler widths of the metal line systems associated with the DLAS). Preliminary reports regarding the amount of neutral hydrogen in such systems deduced from the latest data at redshifts above 3.5 appear to be consistent with these predictions [130]. But a possible problem is the large velocity widths of the metal line systems associated with the highest-redshift DLAS yet reported [131], at $z = 4.4$; if these actually indicate that a massive disk galaxy is already formed at such a high redshift, and if discovery of other such systems shows that they are not rare, that would certainly disfavor CHDM and other $\Omega = 1$ theories with relatively little power on small scales. Other interpretations of such data which would not cause such problems for theories like CHDM are perhaps more plausible, though [132].

One of the best ways of probing early structure formation would be to look at the main light output of the stars of the earliest galaxies, which is redshifted by the expansion of the universe to wavelengths beyond about 5 microns today. Unfortunately, it is not

possible to make such observations with existing telescopes; since the atmosphere blocks almost all such infrared radiation, what is required is a large infrared telescope in space. The Space Infrared Telescope Facility (SIRTF) has long been a high priority, and it would be great to have access to the data such an instrument would produce. In the meantime, an alternative method is to look for the starlight from the earliest stars as extragalactic background infrared light (EBL). Although it is difficult to see this background light directly because our Galaxy is so bright in the near infrared, it may be possible to detect it indirectly through its absorption of TeV gamma rays (via the process $\gamma\,\gamma \to e^+\,e^-$). Of the more than twenty active galactic nuclei (AGNs) that have been seen at ~ 10 GeV by the EGRET detector on the Compton Gamma Ray Observatory, only two of the nearest, Mk421 and Mk501, have also been clearly detected in TeV gamma rays by the Whipple Atmospheric Cerenkov Telescope [133]. Absorption of $\sim$ TeV gamma rays from (AGNs) at redshifts $z \sim 0.2$ has been shown to be a sensitive probe of the EBL and thus of the era of galaxy formation [134].

9 Neutrino mass

There are several experiments which suggest that neutrinos have mass. In particular, the recent announcement of the observation of $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations at the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos suggests that $\delta m^2 \equiv |m(\nu_\mu)^2 - m(\nu_e)^2| \approx 6$ eV^2 [135], and the observation of the angular dependence of the atmospheric muon neutrino deficit at Kamiokande [136] suggests $\nu_\mu \to \nu_\tau$ oscillations are occurring with an oscillation length comparable to the depth of the atmosphere, which requires that the muon and tau neutrinos have approximately the same mass. If, for example, $m(\nu_e) \ll m(\nu_\mu)$, then this means that $m(\nu_\mu) \approx m(\nu_\tau) \approx 2.4$ eV [137,138]. Clearly, discovery of neutrino mass in the few eV range favors CHDM; and, as I mentioned above, this total neutrino mass of about 5 eV is just what seems to be necessary to fit the large scale structure observations [129]. Dividing the mass between two neutrino species results in somewhat lower fluctuation amplitude on the scale of clusters of galaxies because of the longer neutrino free streaming length, which improves agreement between CHDM normalized to COBE and observations of cluster abundance [137].

Of course, one cannot prove a theory since contrary evidence may always turn up. But one can certainly disprove theories. The minimum neutrino mass required by the preliminary LSND result [135] $\delta m^2 = 6$ eV^2 is 2.4 eV. This is too much hot dark matter to permit significant structure formation in a low-Ω universe; for example, in a ΛCHDM model with $\Omega_0 = 0.3$, the cluster number density is more than two orders of magnitude lower than observations indicate [137]. Thus if this preliminary LSND result is correct, it implies a strong lower limit on Ω_0, and a corresponding upper bound on Λ, in ΛCDM models that include light neutrinos.

10   Conclusions

The main issue that I have tried to address is the value of the cosmological density parameter Ω. Strong arguments can be made for $\Omega_0 \approx 0.3$ (and models such as ΛCDM) or for $\Omega = 1$ (for which the best class of models that I know about is CHDM), but it is too early to tell for sure which is right.

The evidence would favor a small $\Omega_0 \approx 0.3$ if (1) the Hubble parameter actually has the high value $H_0 \approx 75$ favored by many observers, and the age of the universe $t_0 \geq 13$ Gyr; or (2) the baryonic fraction $f_b = M_b/M_{tot}$ in clusters is actually $\sim 15\%$, about 3 times larger than expected for standard Big Bang Nucleosynthesis in an $\Omega = 1$ universe. This assumes that standard BBN is actually right in predicting that the density of ordinary matter Ω_b lies in the range $0.009 \leq \Omega_b h^2 \leq 0.02$. High-resolution high-redshift spectra are now providing important new data on primordial abundances of the light isotopes that should clarify the reliability of the BBN limits on Ω_b. If the systematic errors in the ^{4}He data are larger than currently estimated, then it may be wiser to use the deuterium upper limit $\Omega_b h^2 \leq 0.03$, which is also consistent with the value $\Omega_b h^2 \approx 0.024$ indicated by the only clear deuterium detection at high redshift, with the same D/H$\approx 2.4 \times 10^{-5}$ observed in two different low-metallicity quasar absorption systems [100]; this considerably lessens the discrepancy between f_b and Ω_b. Another important constraint on Ω_b will come from the new data on small angle CMB anisotropies — in particular, the height of the first Doppler peak [139], with the latest data consistent with low $h \approx 0.5$ and high $\Omega_b \approx 0.1$.

The evidence would favor $\Omega = 1$ if (1) the POTENT analysis of galaxy peculiar velocity data is right, in particular regarding outflows from voids or the inability to obtain the present-epoch non-Gaussian density distribution from Gaussian initial fluctuations in a low-Ω universe; or (2) the preliminary report from LSND indicating a neutrino mass ≥ 2.4 eV is right, since that would be too much hot dark matter to allow significant structure formation in a low-Ω ΛCDM model.

The statistics of gravitational lensing of quasars is incompatible with large cosmological constant Λ and low cosmological density Ω_0. Discrimination between models may improve as additional examples of lensed quasars are searched for in large surveys such as the Sloan Digital Sky Survey.

It now appears to be possible to measure the deceleration parameter $q_0 = \Omega_0/2 - \Omega_\Lambda$ on very large scales using the objects that may be the best bright standard candles: high-redshift Type Ia supernovae. It is very encouraging that the Perlmutter group [70] now has discovered ~ 25 high-redshift Type Ia supernovae, that other groups are also succeeding in finding such supernovae, and that a theoretical understanding of the empirical correlation between SN Ia light curve shape and maximum luminosity may be emerging. If the high value $q_0 \sim 0.5$ in the preliminary report [70] is right, Ω_Λ is probably small and $\Omega_0 \sim 1$.

The era of structure formation is another important discriminant between these alternatives, low Ω favoring earlier structure formation, and $\Omega = 1$ favoring later formation with many clusters and larger-scale structures still forming today. A particularly critical

test for models like CHDM is the evolution as a function of redshift of Ω_{gas} in damped Lyα systems.

Reliable data on all of these issues is becoming available so rapidly today that there is reason to hope that a clear decision between these alternatives will be possible within the next few years.

What if the data ends up supporting what appear to be contradictory possibilities, e.g. large Ω_0 *and* large H_0? Exotic initial conditions (e.g. "designer" primordial fluctuation spectra) or exotic dark matter particles beyond the simple "cold" vs. "hot" alternatives (e.g. decaying intermediate mass neutrinos) could increase the space of possible inflationary theories somewhat. But it may ultimately be necessary to go outside the framework of inflationary cosmological models and consider models with large scale spatial curvature, with a fairly large Λ as well as large Ω_0. This seems particularly unattractive, since in addition to implying that the universe is now entering a final inflationary period, it means that inflation did not happen at the beginning of the universe, when it would solve the flatness, horizon, monopole, and structure generation problems. Therefore, along with most cosmologists, I am rooting for the success of inflation-inspired cosmologies, with $\Omega_0 + \Omega_\Lambda = 1$. With the new upper limits on Λ from gravitational lensing of quasars, number counts of elliptical galaxies, and high-redshift Type Ia supernovae, this means that the cosmological constant is probably too small to lengthen the age of the universe significantly. So I am hoping that when the dust finally settles, H_0 and t_0 will both turn out to be low enough to be consistent with General Relativistic cosmology. But of course the universe is under no obligation to live up to our expectations.

References

[1] This is a considerably revised version of my article in *Particle and Nuclear Astrophysics and Cosmology in the Next Millenium*, eds. E. Kolb and R. Peccei (Singapore: World Scientific, 1995), pp. 85-98. A popular version is J. Roth and J.R. Primack, *Sky and Telescope*, 91 (1), 20 (January 1996).

[2] W.L. Freedman et al., Nature 371, 757 (1994).

[3] G.R. Blumenthal, S.M. Faber, J.R. Primack, and M.J. Rees, Nature 311, 517 (1984); Erratum: 313, 72 (1985). M. Davis, G. Efstathiou, C.S. Frenk, and S.D.M. White, Astrophys. J. 292, 371 (1985).

[4] P.J.E. Peebles, Astrophys. J. 284, 439 (1984).

[5] L.A. Kofman, N.Y. Gnedin, and N.A. Bahcall, Astrophys. J. 413, 1 (1993); R. Cen, N.Y. Gnedin, and J.P. Ostriker, Astrophys. J. 417, 387 (1993); R. Cen and J.P. Ostriker, Astrophys. J. 429, 4 (1994).

[6] S.A. Bonometto and R. Valdarnini, Phys. Lett. 103A, 369 (1984); L.Z. Fang, S.X. Li, S.P. Xiang, Astron. Astrophys. 140, 77 (1984); A. Dekel and S.J. Aarseth, Astroph. J. 283, 1 (1984); Q. Shafi and F.W. Stecker, Phys. Rev. Lett. 53, 1292 (1984).

[7] M. Davis, F. Summers, and D. Schlegel, Nature 359, 393 (1992).

[8] A. Klypin, J. Holtzman, J.R. Primack, and E. Regős, Astrophys. J. 416, 1 (1993).

[9] C.J. Copi, D.N. Schramm, M.S. Turner, Science 267, 192 (1995). Cf. also R.A. Malaney and G.J. Mathews, Phys. Rep. 229, 145 (1993); L.M. Krauss and P.J. Kernan, Phys. Lett. B, 347, 347 (1995); N. Hata et al., Phys. Rev. Lett. 75, 3977 (1995); C.J. Copi, D.N. Schramm, and M.S. Turner, Phys. Rev. Lett. 75, 3981 (1995); B.D. Fields and K. Olive, Phys. Lett. B, 368, 103 (1996).

[10] A. Dressler et al., Astrophys. J. 313, L37 (1987); D. Lynden-Bell et al., Astrophys. J. 326, 19 (1988).

[11] See e.g. S. Olivier, J.R. Primack, G.R. Blumenthal, and A. Dekel, Astroph. J. 408, 17 (1993); A. Klypin and G. Rhee, Astroph. J. 428, 399 (1994); and references therein.

[12] S.D.M. White, G. Efstathiou, and C.S. Frenk, Mon. Not. R. Astron. Soc. 262, 1023 (1993).

[13] J. Holtzman, Astrophys. J. Supp. 71, 1 (1989).

[14] E.L. Wright, Astrophys. J. 396, L13 (1992).

[15] J.A. Holtzman and J.R. Primack, Astrophys. J. 405, 428 (1993). These results were presented (in J.R. Primack and J.A. Holtzman, in *Gamma Ray — Neutrino Cosmology and Planck Scale Physics: Proc. 2nd UCLA Conf.*, February 1992, D. Cline, ed. (World Scientific, 1993), pp. 28-44) before the anouncement of the COBE discovery. Cf. also T. van Dalen and R.K. Schaefer, Astrophys. J. 398, 33 (1992); R.K. Schaefer and Q. Shafi, Phys. Rev. D 47, 1333 (1993).

[16] M. Bolte and C.J. Hogan, Nature, 376, 399 (1995).

[17] Note however that while R. Jimenez et al., astro-ph/9602132, Mon. Not. R. Astron. Soc. in press (1996) gives a best estimate for the ages of the oldest GCs of 13.5±2 Gyr, they also give a lower limit of 9.7 Gyr. A new analysis by M. Salaris, S. Degl'Innocenti, A. Weiss, astro-ph/9603092 (1996), also gives 13 Gyr as the best estimate of the ages of the three old GCs that they studied (M15, M68, M92), and says that a lower age is possible.

[18] B. Chaboyer, Astrophys. J. 444, L9 (1995) includes a table showing the effects on the GC ages of more than 20 possible changes in input physics and abundances; a Monte Carlo study (B. Chaboyer et al. 1995, astro-ph/9509115) of variations in these parameters gives a 95% CL lower limit on the ages of the oldest GCs of 12 Gyr.

[19] X. Shi, Astrophys. J. 446, 637 (1995).

[20] R.A. Malaney, G.J. Mathews, and D.S.P. Dearborn, Astrophys. J. 345, 169 (1989); G.J. Mathews and D.N. Schramm, Astrophys. J. 404, 468 (1993).

[21] D.E. Winget et al., Astrophys. J. 315, L77 (1987).

[22] D.A. VandenBerg et al., Astron. J. 100, 445 (1990); ibid., 102, 1043 (1991).

[23] J.W. Yuan, Astron. Astrophys. 224, 108 (1989); 261, 105 (1992). V. Weidemann, Ann. Rev. Astron. Astrophys. 28, 103 (1990). M.A. Wood, Astrophys. J. 386, 539 (1992). M. Hernanz et al., Astrophys. J. 434, 652 (1994).

[24] T.D. Oswalt, J.A. Smith, and M.A. Wood, Nature, submitted.

[25] A. Sandage, in *Practical Cosmology: Inventing the Past*, 23rd Sass Fee lectures, ed. B. Binggeli and R. Buser (Berlin: Springer, 1995). A. Sandage and G.A. Tammann, in *Advances in astrofundamental physics, Proc. Int. School of Physics "D. Chalonge"*, eds. N. Sanchez and A. Zichichi (World Scientific, 1995).

[26] G.A. Tammann et al., astro-ph/9603076, to be pub. in *Science with the HST - II* (1996).

[27] J.P. Huchra, Science, 256, 321 (1992). S. Van den Bergh, Science 258, 421 (1992).

[28] G.H. Jacoby et al., Publ. Astron. Soc. Pac., 104, 599 (1992). R. Ciradullo, G.H. Jacoby, and J.L. Tonry, Astrophys. J. 419, 479 (1993).

[29] M. Fukugita, C.J. Hogan, and P.J.E. Peebles, Nature 366, 309 (1993).

[30] R.C. Kennicutt, W.L. Freedman, and J.R. Mould, Astron. J. 110, 1476 (1995).

[31] M. Rowan-Robinson, *The Cosmic Distance Ladder* (1985).

[32] M.J. Pierce et al., Nature 371, 385 (1994).

[33] S. van den Bergh, astro-ph/9509117 (1995); D. Branch et al., astro-ph/9604006 (1996). Cf. B. Schaefer, Astrophys. J. 459, 438 (1996).

[34] M.M. Phillips, Astrophys. J. 413, L105 (1993); M. Hamuy et al., Astron. J. 109, 1 (1995). A.G. Riess, W.H. Press, and R.P. Kirshner, Astrophys. J. 438, L17 (1995); they get $h = 0.67 \pm 0.07$.

[35] G.A. Tammann and S. Sandage, Astrophys. J. 452, 16 (1995).

[36] J. Tonry, Astrophys. J. 373, L1 (1991).

[37] W.D. Arnet, D. Branch, and J.C. Wheeler, Nature 314, 337 (1985); D. Branch, Astroph. J. 392, 35 (1992). For more recent results see A. Fisher et al., Astrophys. J. 447, L73; S. van den Bergh, astro-ph/9509117, Astrophys. J. in press.

[38] B. Leibundgut and P.A. Pinto, Astroph. J. 401, 49 (1992). But cf. T.E. Vaughan et al., Astrophys. J. 439, 558 (1995).

[39] P. Nugent, D. Branch, et al., Phys. Rev. Lett. 75, 394; Erratum 75, 1874 (1995). Reviews: D. Branch and A.M. Khokhlov, Phys. Rep. 256, 53 (1995); D. Branch et al., astro-ph/9601006, to appear in *Proc. NATO Advanced Studies Institute on Thermonuclear Supernovae*, Aiguablava, Spain, June, 1995, eds. R. Canal, P. Ruiz-Lapuente, and J. Isern (1996).

[40] B.P. Schmidt, R.P. Kirschner, and R.G. Eastman, Astrophys. J. 395, 366 (1992).

[41] B.P. Schmidt et al., Astrophys. J. 432, 42 (1994). See also Robert Kirschner's Varenna lectures.

[42] M. Best and R. Wehrse, Astron. Astroph. 284, 507 (1994). P. Ruiz-Lapuente et al., Astrophys. J. 439, 60 (1995).

[43] M. Birkinshaw, J.P. Hughes, and K.A. Arnoud, Astrophys. J. 379, 466 (1991).

[44] M. Birkinshaw and J.P. Hughes, Astrophys. J. 420, 33 (1994).

[45] K. Yamashita, in *New Horizon of X-ray Astronomy — First Reslts from ASCA*, eds. F. Makino and T. Ohashi (Universal Academy Press, Tokyo, 1994), p. 279.

[46] T.M. Wilbanks et al., Astrophys. J. 427, L75 (1994), and parallel session talk at Snowmass 94.

[47] Y. Rephaeli, Ann. Rev. Astron. Astrophys 33, 541 (1995).

[48] J. Pelt et al., Astron. Astroph. 286, 775 (1994).

[49] W.H. Press, G.B. Rybicki, and J.N. Hewitt, Astrophys. J. 385, 416 (1992).

[50] T. Kundic et al., Astrophys. J. 455, L5 (1995).

[51] G.F.R.N. Rhee, Nature 350, 211 (1991).

[52] D.H. Roberts et al., Nature 352, 43 (1991).

[53] H. Danle, S.J. Maddox, and P.B. Lilje, Astrophys. J. 435, L79 (1994). Also further analyses in prep.

[54] N.A. Grogan and R. Narayan, Astrophys. J. in press (1996).

[55] J. Mould et al., Astrophys. J. 449, 413 (1995). Cf. also N.R. Tanvir et al., astro-ph/9509160, which uses new HST Cepheids in M96 to calibrate early-type galaxies and deduce a distance to the Coma cluster, leading to $H_0 = 69 \pm 8$.

[56] E.L. Turner, R. Cen, and J.P. Ostriker, Astron. J. 103, 1427 (1992); X.-P. Wu et al., preprint (1995).

[57] Y. Suto, T. Suginohara, and Y. Inagaki, Prog. Theor. Phys. 93, 839 (1995).

[58] A. Kim et al., astro-ph/9602123, to appear in *Thermonuclear Supernovae (NATO ASI)*, eds. R. Canal, P. Ruiz-LaPuente, and J. Isern (1996).

[59] M. Sasaki, et al. Phys. Lett. B 317, 510 (1993). B. Ratra and P.J.E. Peebles, Astrophys. J. 432, L5 (1994); Phys. Rev. D 52, 1837 (1995). M. Kamionkowski et al., Astrophys. J. 434, L1 (1994). M. Sasaki, T. Tanaka, and K. Yamamoto, Phys. Rev. D 51, 2979 (1995). M. Bucher, A.S. Goldhaber, and N. Turok, Phys. Rev. D 52, 3314 (1995).

[60] O. Lahav, P. Lilje, J.R. Primack, and M.J. Rees, Mon. Not. R. Astron. Soc. 251, 128 (1991).

[61] S.M. Carroll, W.H. Press, and E.L. Turner, Ann. Rev. Astron. Astrophys, 30, 499 (1992).

[62] D. Maoz and H.W. Rix, Astrophys. J. 416, 425 (1993).

[63] C. Kochanek, Astrophys. J. 419, 12 (1993).

[64] C.S. Kochanek, astro-ph/9510077, Astrophys. J. in press (1996).

[65] C. Steidel, M. Dickinson, and S.E. Persson, Astrophys. J. 437, L75 (1994); S. Lilly et al., Astrophys. J. 455, 108 (1995); D. Schade et al., astro-ph/9604032 (1996).

[66] E. Bunn and N. Sugiyama, Astrophys. J. 446, 49 (1995). However, this limit may be stronger than is justified by a more complete analysis of the COBE data (E. Wright and K. Gorski, private communications).

[67] S. Driver et al., astro-ph/9511141, Astrophys. J. in press (April 1996).

[68] A. Klypin, J.R. Primack, and J. Holtzman, astro-ph/9510042, Astrophys. J. in press (July 1996).

[69] G. Goldhaber et al., Nucl. Phys. B, S38, 435 (1995). A. Goobar and S. Perlmutter, Astrophys. J. 450, 14 (1995). S. Perlmutter et al., Astrophys. J. 440, L41 (1995).

[70] S. Perlmutter, astro-ph/9602122, to appear in *Thermonuclear Supernovae (NATO ASI)*, eds. R. Canal, P. Ruiz-Lapuente, and J. Isern (1996).

[71] A. Dekel, Ann. Rev. Astron. Astroph. 32, 371 (1994). M.A. Strauss and J.A. Willick, Phys. Rep. 261, 271 (1995). The latest POTENT results on the power spectrum of mass fluctuations are in S. Zaroubi et al., astro-ph/9603068 (1996) and T. Kolatt and A. Dekel, astro-ph/9512132 (1995).

[72] A. Dekel and M.J. Rees, Astrophys. J. 422, L1 (1994).

[73] A. Nusser and A. Dekel, Astrophys. J. 405, 437 (1993). Similar constraints have been derived using a perturbative technique by F. Bernardeau, R. Juskiewicz, A. Dekel, and F.R. Bouchet, Mon. Not. R. Astron. Soc. 274, 20 (1995).

[74] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton Univ. Press, 1992), esp. §20.

[75] M. Davis and P.J.E. Peebles, Astrophys. J. 267, 465 (1983).

[76] R. Somerville, M. Davis, and J.R. Primack, astro-ph/9604041, Astrophys. J. submitted (1996).

[77] H.J. Mo, Y.P. Jing, and G. Börner, Mon. Not. R. Astron. Soc. 264, 825 (1993). W. Zurek *et al.*, Astrophys. J. 431, 559 (1994).

[78] K.B. Fisher, et al., Mon. Not. R. Astron. Soc. 267, 927 (1994).

[79] R.O. Marzke, M.J. Geller, L.N. da Costa, and J.P. Huchra, Astron. J. 110, 477 (1995).

[80] R. Somerville, J.R. Primack, and R. Nolthenius, astro-ph/9604051, Astrophys. J. submitted (1996).

[81] R. Kirschner, A. Oemler, and P. Schechter, AJ 84, 951 (1979).

[82] J.P. Huchra and M.J. Geller, Astrophys. J. 257, 423 (1982).

[83] M. Ramella, M.J. Geller, and J.P. Huchra, Astrophys. J. 344, 57 (1989).

[84] R. Nolthenius, A. Klypin, and J.R. Primack, astro-ph/9410095, Astrophys. J., in press (1996).

[85] R.G. Carlberg and H.M.P. Couchman, Astrophys. J. 340, 47 (1989).

[86] R. Nolthenius, A. Klypin, and J.R. Primack, Astrophys. J. 422, L45 (1994).

[87] P.J.E. Peebles, Astrophys. J. 344, 53 (1989); 362, 1 (1990); 429, 43 (1994).

[88] E. Branchini and R.G. Carlberg, Astrophys. J. 434, 37 (1994).

[89] A.M. Dunn and R. Laflamme, Astrophys. J. 443, L1 (1995).

[90] E.J. Shaya, P.J.E. Peebles, and R.B. Tully, Astrophys. J. 454, 15 (1995).

[91] D. Zaritsky et al., Astrophys. J. 405, 464 (1993); D. Zaritsky and S.D.M. White, Astrophys. J. 435, 599 (1994).

[92] D. Zaritsky, private communication (1995).

[93] J. Dubinski, J.C. Mihos, and L. Hernquist, astro-ph/9509010, Astrophys. J. in press (1996).

[94] J.A. Tyson et al., Astrophys. J. 281, L59 (1984); cf. I. Kovner and M. Milgrom, Astrophys. J. 321, L113 (1987).

[95] J. Mould et al., Mon. Not. R. Astron. Soc. 271, 31 (1994).

[96] J. Villumsen, astro-ph/9507007 (1995).

[97] B. Little and S. Tremaine, Astrophys. J. 320, 493 (1987).

[98] C.S. Kochanek, astro-ph/9505068, submitted to Astrophys. J. (1995).

[99] A. Songaila et al., Nature 368, 599 (1994); R.F. Carswell et al. Mon. Not. R. Astron. Soc. 268, L1 (1994).

[100] D. Tytler and X. Fan, Bull. Am. Astron. Soc. 26, 1424 (1994); D. Tytler, X. Fan, and S. Burles, astro-ph/9603069, submitted to Nature (1996). S. Burles and D. Tytler, astro-ph/9603070, submitted to Science (1996).

[101] M. Rugers and C.J. Hogan, Astrophys. J. 459, L1 (1996). cf. C.J. Hogan, astro-ph/9512003 (1995).

[102] E.J. Wampler et al., astro-ph/9512084, submitted to Astron. Astroph. (1995).

[103] R.F. Carswell et al., Mon. Not. R. Astron. Soc. 278, 506 (1996).

[104] S.D.M. White and C.S. Frenk, Astrophys. J. 379, 52 (1991). S.D.M. White et al. Nature 366, 429 (1993). Cf. Lubin et al., Astrophys. J. 460, 10 (1996); A.E. Evrard, C.A. Metzler, and J.F. Navarro, FERMILAB-Pub-95/337-A (1995); S. Schindler, Astron. Astroph. 305, 756 (1996). Review: G. Steigman and J.E. Felten, Space Science Reviews 74, 245 (1995).

[105] D.A. White and A.C. Fabian, Mon. Not. R. Astron. Soc., 273, 72 (1995). Cf. R. Mushotzky et al., in *Dark Matter*, ed. S.S. Holt and C.L. Bennett, AIP Conf. Proc. 336, 231 (1995); D.A. Buote and C.R. Canizares, Astrophys. J. 457, 565 (1996); S. Bardelli et al., Astron. Astroph. 305, 435 (1996). However, recent ASCA data on several rich clusters imply lower baryon fractions, less than 0.1 for $h = 0.5$ (R. Mushotzky private communication, March 1996), which may call into question the "fair sample" hypothesis.

[106] K.F. Gunn and P.A. Thomas, astro-ph/9510082 (1995) show that the baryon fraction could be overestimated by a factor of 2 or more if the cluster gas is a multiphase medium.

[107] A. Loeb and S. Mao, Astrophys. J. 435, L109 (1994). P. Biermann (private communication) has suggested that such tangled fields might extend throughout clusters; cf. J.E. Felten, in *Clusters, Lensing, and the Future of the Universe*, eds. V. Trimble and A. Reisenegger, ASP Conf. Series 88, 271 (1996).

[108] P.P. Kronberg, Reports on Progress in Physics 57, 325 (1994).

[109] C. Balland & A. Blanchard, astro-ph/9510130 (1995).

[110] N.A. Bahcall and L.M. Lubin, Astrophys. J. 426, 513 (1994). M. Bartelmann and R. Narayan, in *Dark Matter*, College Park, MD, October 1994 (AIP Conference Proceedings 336, 1995) 307.

[111] G. Falhman et al. Astrophys. J. 437, 56 (1994), reviewed in N. Kaiser et al., astro-ph/9407004 (1994). There are several similar examples. However, R.G. Carlberg, H.K.C. Yee, and E. Ellingson, astro-ph/9512087 (1995) show that the velocities of galaxies around 16 galaxy clusters are probably not consistent with such large dark matter halos. This puzzling discrepancy needs further study.

[112] This is shown visually (with accompanying video; high-resolution frames and low-resolution mpeg excerpts from the video can be viewed or downloaded from our WWW page http: //physics.ucsc.edu/groups/cosmology.html) in my group's CHDM simulations in D. Brodbeck et al., Astrophys. J. in press (1996); cf. also A. Klypin, R. Nolthenius, and J.R. Primack, astro-ph/9502062, Astrophys. J. in press (1996).

Observed cluster temperatures and even X-ray luminosities appear to be well fit, according to the first CHDM hydrodynamic cluster simulation, G. Bryan et al., Astrophys. J. 437, L5 (1994).

[113] L. Kofman, A. Klypin, D. Pogosyan, and J.P. Henry, astro-ph/9509145 (1995).

[114] D. Richstone, A. Loeb, and E.L. Turner, Astrophys. J. 393, 477 (1992).

[115] J.O. Burns et al., Astrophys. J. 427, L87 (1994).

[116] J.J. Mohr, A.E. Evrard, D.G. Fabricant, and M.J. Geller, Astrophys. J. 447, 8 (1995).

[117] Y.P. Jing, H.J. Mo, G. Börner, and L.Z. Fang, in *International Workshop on Large Scale Structure in the Universe*, Potsdam, 1994 (World Scientific, 1995); Mon. Not. R. Astron. Soc. 276, 417 (1995).

[118] E.g., D.A. Buote and J.C. Tsai, Astrophys. J., 458, 27 (1996), and J.C. Tsai and D.A. Buote, astro-ph/9510057, Mon. Not. R. Astron. Soc. in press (1996).

[119] J.P. Henry et al., Astrophys. J. 386, 408 (1992). F.J. Castander et al., Nature 377, 39 (1995). H. Ebeling et al., to appear in *Roentgenstrahlung from the Universe*, Wurzberg, Sept. 1995.

[120] C.C. Steidel, M. Dickinson, and S.E. Persson, Astrophys. J. 437, L75 (1994).

[121] G. Kauffmann, Mon. Not. R. Astron. Soc. 274, 161 (1995).

[122] A. Klypin et al., in prep. (1996).

[123] M.G. Haehnelt, Mon. Not. R. Astron. Soc. 265, 727 (1993).

[124] N. Katz, T. Quinn, E. Bertschinger, and J.M. Gelb, Mon. Not. R. Astron. Soc. 270, L71. Cf. D.J. Eisenstein and A. Loeb, Astrophys. J. 443, 11 (1995).

[125] C.C. Steidel et al., astro-ph/9602024 (1996), and M. Giavalisco, C.C. Steidel, and P.D. Macchetto, astro-ph/9603062 (1996).

[126] A.M. Wolfe, in *Relativistic Astrophysics and Particle Cosmology*, eds. C.W. Ackerlof, M.A. Srednicki (New York, New York Academy of Science, 1993), p. 281.

[127] K.M. Lanzetta, Publ/ Astron. Soc. Pac., 105, 1063, (1993); K.M. Lanzetta, A.M. Wolfe and D.A. Turnshek, Astrophys. J. 440, 435 (1995).

[128] H.J. Mo and J. Miralda-Escude, Astrophys. J. 430, L25 (1994); G. Kauffmann and S. Charlot, Astrophys. J. 430, L97 (1994); C.-P. Ma and E. Bertschinger, Astrophys. J. 434, L5 (1994).

[129] A. Klypin, S. Borgani, J. Holtzman, and J.R. Primack, Astrophys. J. 444, 1 (1995).

[130] L.J. Storrie-Lombardie, R.G. McMahon, M.J. Irwin, and C. Hazard, Astrophys. J. 427, L13-16 (1994); —-, *Proc. ESO Workshop on QSO Absorption Lines*, ed. G. Meylan (Springer, 1995). Newer data suggest that $\Omega_{gas} \approx 3 \times 10^{-3}$ for $z \approx 2 - 3.5$, and either declining or at most remaining constant for higher $z \sim 4 - 4.5$ (private communications from Lisa Storrie-Lombardi and Art Wolfe, December 1995).

[131] L.M. Lu et al., Astrophys. J. 457, L1 (1996).

[132] M. Haehnelt, M. Steinmetz, and M. Rauch, astro-ph/9512118 (1996).

[133] J. Quinn et al., Astrophys. J. 456, L83 (1996); M.S. Schubnell et al., astro-ph/9602068, Astrophys. J. in press (1996).

[134] D. MacMinn and J.R. Primack, in *TeV Gamma Ray Astrophysics*, ed. Heinz Völk and F. Aharonian, Space Science Reviews, 75, 413 (1996). Cf. Ground Based Gamma-Ray Astronomy, in *Particle and Nuclear Astrophysics and Cosmology in the Next Millenium*, eds. E. Kolb and R. Peccei (Singapore: World Scientific, 1995), p. 295.

[135] D.O. Caldwell, in *Trends in Astroparticle Physics*, Stockholm, Sweden 22-25 September 1994, eds. L. Bergstrom, P. Carlson, P.O. Hulth and N. Snellman, Nucl. Phys. B, Proc. Suppl., 43, 126 (1995). C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995). In the latest preprint from LSND, available at http://nu1.lampf.lanl.gov~lsnd/, the 1995 data strengthens the statistics implying oscillations, but perhaps weakens the case for $\delta m^2 = 6$ eV^2.

[136] Y. Fukuda, Phys. Lett. B 335, 237 (1994).

[137] J.R. Primack, J. Holtzman, A. Klypin, and D.O. Caldwell, in *Trends in Astroparticle Physics*, Stockholm, Sweden 22-25 September 1994, eds. L. Bergstrom, P. Carlson, P.O. Hulth and N. Snellman, Nucl. Phys. B, Proc. Suppl., 43, 133 (1995), and Phys. Rev. Lett. 74, 2160 (1995); J.R. Primack, J. Holtzman, and A. Klypin, Moriond 1995, in press. Cf. D.Yu. Pogosyan and A.A. Starobinsky, in *International Workshop on Large Scale Structure in the Universe*, Potsdam, 1994 (World Scientific, 1995) and Astrophys. J. 447, 465 (1995); K.S. Babu, R.K. Schaefer, and Q. Shafi, Phys. Rev. D53, 606 (1996); A.R. Liddle et al., astro-ph/9511057 (1995).

[138] However, an inverted neutrino mass hierarchy with ν_e the most massive appears to be required if all the current experimental evidence for neutrino mass (solar and atmospheric neutrino deficits, and LSND) are valid, and r-process nucleosynthesis (responsible for production of the heavy elements) takes place in the hot ν bubble a few hundred km above the neutron star in Type II supernovae; see G.M. Fuller, J.R. Primack, and Y. Qian, Phys. Rev. D 52, 1288 (1995).

[139] See e.g. S. Dodelson, E. Gates, and A. Stebbins, astro-ph/9509147 (1995); G. Jung-man et al., astro-ph/9512139 (1995); M. Tegmark, astro-ph/9601077, Astrophys. J. Lett. in press (1996).